

Incorporating Estimation Risk in Portfolio Choice

Jenke R. ter Horst*, Frans A. de Roont[†] and Bas J.M. Werker[‡]

June 11, 2000

Abstract

We propose an adjustment in mean-variance portfolio weights to incorporate uncertainty caused by the fact that, in general, we have to use *estimated* expected returns. The adjustment amounts to using a higher pseudo risk-aversion rather than the actual risk-aversion. The difference between the actual and the pseudo risk-aversion depends on the sample size, the number of assets in the portfolio, and the curvature of the mean-variance frontier. Applying the adjustment to international portfolios, we show that the adjustments are nontrivial for G5 country portfolios and that they are even more important when emerging markets are included. We also show that, in the case of time-varying expected country returns, our adjustment implies a significantly smaller variability in portfolio weights than is commonly believed.

*Department of Finance and CentER, Tilburg University. E-mail: J.R.TerHorst@kub.nl

[†]Financial Management Department, Erasmus University Rotterdam, and CEPR. E-mail: F.Roon@fbk.eur.nl

[‡]Department of Finance and CentER, Tilburg University. E-mail: Werker@kub.nl

[§]Helpful comments of Bertrand Melenberg and Theo Nijman are gratefully acknowledged.

1 Introduction

A common problem in portfolio selection is the fact that the necessary parameter values are usually unknown and have to be estimated. For instance, when implementing the mean-variance efficient portfolios introduced by Markowitz (1959), the mean returns and (co)variances are usually estimated from available data. This may lead to suboptimal portfolios. Since mean-variance efficient portfolio weights are very sensitive to the level of the expected returns and since it is well-known that uncertainty in the estimated mean returns is higher than in the estimated (co)variances, it is especially the uncertainty in the mean returns that has a large influence on portfolio weights.

Previous papers have tried to come up with estimates of the mean returns that improve upon the sample average using for instance shrinkage or Stein estimators (Jobson, Korkie, and Ratti (1979), Jorion (1985, 1986, 1991)). These estimators shrink the means towards a common value. Alternatively, Jorion (1991) uses so-called CAPM estimators in which the means are assumed to be proportional to their beta relative to the market portfolio. Other papers have also dealt with the problem of estimation risk using a Bayesian approach. Examples are Klein and Bawa (1976), Barry (1974), Chen and Brown (1983), and Alexander and Resnick (1985).

A disadvantage of both shrinkage and CAPM-based estimators is that they presuppose a strong prior belief with respect to expected returns, such as that there is a common value for the means or that expected returns can be fully explained by their market beta. In this paper we take the uncertainty in mean returns as given and propose an adjustment in mean-variance efficient portfolio weights that incorporates this uncertainty or estimation risk. Using the loss in expected utility when implementing a suboptimal portfolio, we show that investors can easily incorporate uncertainty in the mean returns by basing their mean-variance efficient portfolio on a pseudo risk-aversion rather than their actual risk-aversion. The pseudo risk-aversion is always higher than the actual risk aversion and the difference between the two depends on the number of assets under consideration, the sample size, and the efficient set constants. As is to be expected, the difference between the pseudo risk-aversion and the actual risk aversion is increasing in the number of assets included in the portfolio and decreasing in the sample size. In the case where there are short sales constraints, the pseudo risk-aversion is calculated in a similar way as in the case where there are no constraints, but the adjustment is based only on the assets for which the constraints are

not binding. Finally, when returns are predictable from a set of observed instruments, the adjustment also depends on the values taken by those instruments.

We illustrate the effect of estimation risk for international asset portfolios based on either the G5 countries or on the G5 countries plus a number of emerging markets. We show that the difference between the pseudo risk-aversion and the actual risk aversion can be sizable even for investors that wish to invest in the G5 countries only. Using a sample of 25 years of monthly data, the difference in expected utility between the portfolio based on the actual risk-aversion and the optimal pseudo risk-aversion translates into an annual equivalent risk premium of about 0.55 percent. This premium increases to 6.7 percent when only five years of monthly data are available.

The effects of estimation risk are even more pronounced in the case where emerging markets are included. In this case, the difference between the actual and pseudo risk-aversion increases dramatically with a corresponding strong effect on the optimal portfolio weights. This effect is a combined result of the increase in the number of assets and the greater uncertainty in the mean returns of emerging markets as reflected in the efficient set constants, and occurs with and without short sales constraints. We also investigate the effect of incorporating estimation risk when expected returns can be predicted from a set of common instruments. We show that there is less variability in the optimal portfolio weights because of the instruments than is commonly believed, if the estimation risk in the predictive regressions is taken into account.

Finally, simulations confirm that uncertainty in the expected returns is indeed much more important than uncertainty in the covariances, for all sample sizes and for all risk aversions considered. As can be expected, the uncertainty in covariances becomes more important as the risk aversion increases, but the magnitude of the loss in expected utility that results from this uncertainty remains small.

The plan of this paper is as follows. Section 2 shows how estimation risk can be incorporated in mean-variance efficient portfolios by using a pseudo risk-aversion coefficient. Section 3 describes the data and Sections 4 through 7 discuss the effect of estimation risk for international asset portfolios. The paper ends with a summary and concluding remarks.

2 Incorporating estimation risk in mean-variance efficient portfolios

2.1 Estimation risk in the *i.i.d.* case

Suppose that an investor has a menu of K different assets from which he chooses his portfolio. The returns on these assets are given by the K -vector R_t , and are assumed to be *i.i.d.* and normally distributed with mean vector μ and covariance matrix Σ . Since returns are normally distributed, the investor chooses his portfolio w to maximize

$$w'\mu - \frac{1}{2}\gamma w'\Sigma w, \quad (1)$$

subject to $w'\iota = 1$, with ι a K -vector of ones, and γ the risk aversion of the investor. It is well known that the optimal portfolio for this investor is given by

$$w^* = \gamma^{-1}\Sigma^{-1}(\mu - \eta\iota), \quad (2)$$

where η is the expected return on the zero-beta portfolio associated with w^* .

In characterizing mean-variance efficient portfolios that satisfy (2) it is useful to define the efficient set constants:

$$A = \iota'\Sigma^{-1}\iota, \quad (3a)$$

$$B = \iota'\Sigma^{-1}\mu, \quad (3b)$$

$$C = \mu'\Sigma^{-1}\mu. \quad (3c)$$

Using these constants it is straightforward to show that the zero-beta rate η can be written as a function of γ : $\eta = (B - \gamma)/A$.

In practice, the parameters μ and Σ are not known of course, but have to be estimated from the data. We assume that the uncertainty in $\hat{\Sigma}$ is small and can be neglected and we will focus on the estimation error in $\hat{\mu}$. Our simulation results in Section 7 show that this is a valid presumption. Based on the estimated mean returns $\hat{\mu}$, suppose that the investor chooses his mean-variance efficient portfolio analogous to (2) as

$$\hat{w}(\alpha) = \alpha^{-1}\Sigma^{-1}(\hat{\mu} - \rho\iota). \quad (4)$$

We refer to this parameter α as the *pseudo risk-aversion*. A naive investor would choose his portfolio by choosing $\alpha = \gamma$. We show that, due to the

uncertainty in estimated mean returns, this is not optimal. The zero-beta rate ρ depends on the pseudo risk-aversion α and the estimated efficient set constants \hat{A} , \hat{B} , and \hat{C} in the same way as η depends on γ , A , B , and C .¹

Since the portfolio $\hat{w}(\alpha)$ depends on the estimated mean returns $\hat{\mu}$ rather than the true parameters μ , it will in general not be equal to the optimal portfolio w^* in (2). Using the suboptimal portfolio $\hat{w}(\alpha)$ yields a loss in utility which, using (1), is equal to

$$L(\alpha) = (w^{*\prime}\mu - \hat{w}(\alpha)'\mu) - \frac{1}{2}\gamma(w^{*\prime}\Sigma w^* - \hat{w}(\alpha)'\Sigma\hat{w}(\alpha)), \quad (5)$$

and the expected loss equals

$$\delta(\alpha) = E[L(\alpha)]. \quad (6)$$

We propose to choose the pseudo risk-aversion α in such a way that the expected loss δ will be minimized, i.e.:

$$\alpha^* = \arg \min_{\{\alpha\}} \delta(\alpha) = \gamma \left(1 + \frac{A}{AC - B^2} \frac{K - 1}{T} \right), \quad (7)$$

where the last equality is derived in the appendix. The optimal value α^* has an obvious interpretation. Since both A and $AC - B^2$ are always positive, the adjustment factor is at least 1, and α^* is always larger than or equal to the actual risk-aversion γ . The fact that the pseudo risk-aversion exceeds the actual risk aversion reflects the higher uncertainty that is caused by using the estimated expected returns $\hat{\mu}$ rather than the true expected returns. Since this uncertainty induces a portfolio that is actually more risky than if the true parameters were known, the investor wants to adjust his portfolio for this by using a higher pseudo risk-aversion and therefore a less risky portfolio. Basically, in using a higher pseudo risk-aversion, the investor selects a portfolio that is closer to the Global Minimum Variance portfolio.

The adjustment factor increases as the number of assets under consideration, K , increases. This reflects the fact that as the number of assets increases, the number of parameters in μ increases, implying a higher level of uncertainty. As the sample size T increases, the estimate $\hat{\mu}$ of μ becomes more precise and the adjustment factor decreases, as is to be expected. Finally, it is straightforward to show that the term $A/(AC - B^2)$ is proportional

¹The estimated parameters depend on $\hat{\mu}$ instead of μ : $\hat{A} = A = \iota'\Sigma^{-1}\iota$, $\hat{B} = \iota'\Sigma^{-1}\hat{\mu}$, and $\hat{C} = \hat{\mu}'\Sigma^{-1}\hat{\mu}$.

to the second derivative of the efficient portfolio's variance with respect to the expected portfolio return. Therefore, this term reflects the curvature of the mean-variance frontier. A high curvature implies that small changes in the expected return of efficient portfolios imply big changes in the corresponding volatility. Stated differently, a large value of $A/(AC - B^2)$ implies that estimation error in the expected returns can be very costly in volatility terms. The high pseudo risk-aversion α^* neutralizes this effect.

2.2 Including short sales constraints

The previous section showed that investors that are unrestricted in their portfolio holdings can account for estimation risk in expected returns by choosing an efficient portfolio based on the pseudo risk-aversion in (7). When investors face short sales constraints, the portfolio problem becomes

$$\begin{aligned} \max_w \quad & w' \mu - \frac{1}{2} \gamma w' \Sigma w, \\ \text{s.t. } \quad & w' \iota = 1 \text{ and } w \geq 0, \end{aligned} \tag{8}$$

where the inequality applies componentwise. In this case, the optimal portfolio is given by

$$w^* = \gamma^{-1} \Sigma^{-1} (\mu - \eta \iota - \lambda), \tag{9}$$

where λ is the vector of Kuhn-Tucker multipliers for the restrictions that the portfolio weights are nonnegative. Denote by $R_t^{(\gamma)}$ the $K^{(\gamma)}$ -dimensional subset of the assets in R_t for which the short sales constraints are not binding. The superscript (γ) refers to this subset. It is straightforward to show that the mean-variance efficient portfolio in equation (9) is equal to the mean-variance efficient portfolio without short sales constraints of the assets in $R_t^{(\gamma)}$ only (see, e.g., Markowitz (1985)). Thus, ordering the portfolio weights in w^* as $w^* = \left(w^{(\gamma)'} \ 0'_{K-K^{(\gamma)}} \right)'$, such that the short sales constraints are not binding for the first $K^{(\gamma)}$ elements and binding for the last $K - K^{(\gamma)}$ elements, we get that

$$\begin{aligned} w^* &= \begin{pmatrix} w^{(\gamma)} \\ 0_{K-K^{(\gamma)}} \end{pmatrix}, \\ w^{(\gamma)} &= \gamma^{-1} \left(\Sigma^{(\gamma)} \right)^{-1} (\mu^{(\gamma)} - \eta \iota_{K^{(\gamma)}}). \end{aligned} \tag{10}$$

Following the ideas in Markowitz (1985) and DeRoos, Nijman, and Werker (2000), notice that for a given set of K asset returns R_t , there is only a finite number N of subsets with $K^{(\gamma)}$ elements, $K^{(\gamma)} \in \{1, 2, \dots, K\}$. Let $G^{[j]}$ be the set of those values of γ for which the subset of assets for which the short sales constraints in the mean-variance efficient portfolios are not binding is the same, and denote the $K^{[j]}$ -dimensional vector of returns for these assets as $R_t^{[j]}$, i.e., $R_t^{[j]} = R_t^{(\gamma)}$ if and only if $\gamma \in G^{[j]}$. Similarly, each variable or parameter that refers to the set $R_t^{[j]}$ will be denoted with a superscript $^{[j]}$. Since for $\gamma \in G^{[j]}$ the restricted mean-variance efficient frontier of R_t coincides with the unrestricted mean-variance frontier of $R_t^{[j]}$, the mean-variance frontier of R_t with short sales constraints consists of a finite number of parts of the unrestricted mean-variance frontiers of the subsets $R_t^{[j]}$.

To see how estimation risk can be incorporated in the optimal portfolio choice when there are short sales constraints, start from a given segment $G^{[j]}$. When moving downwards along the frontier, the transition point in terms of α between two segments is defined by $\alpha^{[j,j+1]}$ such that $\alpha^{[j,j+1]} \in G^{[j]}$ and $\alpha^{[j,j+1]} \in G^{[j+1]}$. Notice that if $G^{[j+1]}$ is the segment adjacent to $G^{[j]}$ when moving downwards along the frontier, then we have that

$$\alpha^{[j,j+1]} = \alpha_{\max}^{[j]} = \alpha_{\min}^{[j+1]}, \quad (11)$$

with

$$\begin{aligned} \alpha_{\max}^{[j]} &= \max_{\alpha \in G^{[j]}} \alpha, \text{ and} \\ \alpha_{\min}^{[j+1]} &= \min_{\alpha \in G^{[j+1]}} \alpha. \end{aligned}$$

Since for a given segment $G^{[j]}$ the mean-variance efficient portfolios are simply the unrestricted portfolios for $R_t^{[j]}$, it follows from the analysis in the previous section that the value of the pseudo risk-aversion α that minimizes the expected loss δ for this segment is,

$$\begin{aligned} \alpha^{[j]*} &= \arg \min_{\alpha \in G^{[j]}} \delta(\alpha) \\ &= \left\{ \begin{array}{l} \alpha^{[j-1,j]} \text{ if } \frac{\partial \delta(\alpha)}{\partial \alpha} \Big|_{\alpha=\alpha^{[j-1,j]}} > 0, \\ \alpha^{[j,j+1]} \text{ if } \frac{\partial \delta(\alpha)}{\partial \alpha} \Big|_{\alpha=\alpha^{[j,j+1]}} < 0, \\ \gamma \left(1 + \frac{K^{[j]}-1}{T} \frac{A^{[j]}}{B^{[j]2} - A^{[j]}C^{[j]}} \right) \text{ otherwise.} \end{array} \right\}. \end{aligned} \quad (12)$$

Having found the minimum expected loss for each of the N segments, it then follows that the optimal value of the pseudo risk-aversion α^* is given by

$$\alpha^* = \arg \min_{\{j=1,2,\dots,N\}} \delta(\alpha^{[j]*}), \quad (13)$$

where it should be noted that if $\alpha^{[j]*}$ is a transition point between two segments, we have that

$$\delta(\alpha^{[j,j+1]}) = \frac{1}{2}\delta(\alpha_{\max}^{[j]}) + \frac{1}{2}\delta(\alpha_{\min}^{[j+1]}). \quad (14)$$

This is because at a transition point there is an equal probability that either $G^{[j]}$ or $G^{[j+1]}$ is the relevant subset of assets for which the short sales constraints are not binding.

Thus, when there are short sales constraints the optimal value of the pseudo risk-aversion α depends in the same way on the sample size T , the number of assets $K^{[j]}$, and the efficient set constants as in the case where there are no constraints, except that $K^{[j]}$ and the efficient set constants are now defined by the relevant subset of assets for which the short sales constraints are not binding. If the optimal pseudo risk-aversion α^* is a transition point between two segments, then we no longer have a closed form solution for the α^* as in the unrestricted case.

2.3 Estimation risk with return predictability

It is well-known by now that stock returns can be predicted from common instruments such as the dividend yield and the short term interest rate (see, e.g., Ferson and Harvey (1999)). We will assume that expected returns can be predicted from a set of instruments, but that the covariance matrix of the unexpected returns is constant, i.e., returns are conditionally homoskedastic. Suppose that stock returns can be predicted from a set of L instruments z_t , which may include a constant:

$$R_t = \beta z_{t-1} + \varepsilon_t, \quad (15)$$

where β is a $K \times L$ matrix and where the error terms ε_t are assumed to be homoskedastic and normally distributed, $\varepsilon_t \sim N(0, \Omega_{\varepsilon\varepsilon})$. Conditionally on the instruments at time $t-1$, the optimal portfolio at time $t-1$ is then given

by

$$w_{t-1}^* = \gamma^{-1} \Omega_{\varepsilon\varepsilon}^{-1} (\mu_{t-1} - \eta_{t-1} \iota), \quad (16a)$$

$$\text{with } \mu_{t-1} = \beta z_{t-1}. \quad (16b)$$

The zero-beta rate is now a time varying function of the risk aversion γ :

$$\eta_{t-1} = \frac{B_{t-1} - \gamma}{A},$$

where $A = \iota' \Omega_{\varepsilon\varepsilon}^{-1} \iota$, $B_{t-1} = \mu_{t-1}' \Omega_{\varepsilon\varepsilon}^{-1} \iota$, and similarly, $C_{t-1} = \mu_{t-1}' \Omega_{\varepsilon\varepsilon}^{-1} \mu_{t-1}$.

As before, the parameters of interest are unknown to the investor and have to be estimated from the data. We assume again that the estimation error in the (co)variances is small and we neglect this uncertainty. We focus on the estimation error in expected returns, which is now caused by the fact that we have to estimate the regression coefficients β . Let the value of the instruments at time $t - 1$ be given by a specific value $z_{t-1} = z_0$. Analogous to the unconditional case in (4), suppose that the investor chooses his conditionally mean-variance efficient portfolio as

$$\widehat{w}(\alpha)_0 = \alpha^{-1} \Omega_{\varepsilon\varepsilon}^{-1} (\widehat{\beta} z_0 - \rho_0 \iota), \quad (17)$$

where, using obvious notation, the subscript 0 always indicates the value of the variables given that $z_{t-1} = z_0$. Since this portfolio depends on the estimated parameters $\widehat{\beta}$ it will in general be suboptimal, and the loss in expected utility resulting from using $\widehat{w}(\alpha)_0$ rather than the optimal portfolio w_0^* , is equal to

$$L(\alpha)_0 = (w_0^{*'} \mu_0 - \widehat{w}(\alpha)_0' \mu_0) - \frac{1}{2} \gamma (w_0^{*'} \Omega_{\varepsilon\varepsilon} w_0^* - \widehat{w}(\alpha)_0' \Omega_{\varepsilon\varepsilon} \widehat{w}(\alpha)_0), \quad (18)$$

with $\mu_0 = \beta z_0$. Likewise, the expected loss, conditionally on the instruments equals

$$\delta_0(\alpha) = E[L(\alpha)_0]. \quad (19)$$

In the appendix it is shown that the value of α that minimizes δ_0 is

$$\alpha_0^* = \gamma \left(1 + D_0 \frac{A}{AC_0 - B_0^2} \frac{(K-1)}{T} \right), \quad (20)$$

$$\text{with } D_0 = z_0' \left(\frac{1}{T} \sum_{t=1}^T z_t z_t' \right)^{-1} z_0.$$

This solution α_0^* generalizes (7) in a straightforward way. Because expected returns depend on the specific value z_0 that z_{t-1} takes, we first of all have that the efficient set variables B_0 and C_0 also depend on the specific value of z_0 . Apart from this, a second adjustment relative to unconditional case has to be made through the term D_0 , which is the inner product of z_0 , weighted by the empirical second moment matrix of z_{t-1} . In the particular case where there is only one instrument which is a constant, i.e., $z_t = 1, \forall t$, (20) reduces to the unconditional case in (7), implying that $\alpha_0^* = \alpha^*$.

3 Data

We use a dataset that contains monthly returns on stock indices for the G5 countries as well as monthly returns on three emerging market indices. The data for the G5 countries are for the period January 1974 until December 1998 and for the emerging markets for the period January 1989 until December 1998. The G5 stock indices are the MSCI indices for the US, France, Germany, Japan, and the United Kingdom. The emerging market indices are the indices for Latin America, Southeast Asia, and the Middle East/Europe. These indices are from the Emerging Markets Data Base (EMDB) of the International Finance Corporation (IFC). The indices for the emerging markets are the IFC Investable indices and therefore they represent stock portfolios that are obtainable for U.S. investors. All data are from Datastream. All returns are monthly unhedged U.S. Dollar returns.

Table 1 contains summary statistics for the returns on the G5 indices as well as the emerging market indices. These summary statistics present some common features of international stock returns. Monthly returns on the G5 indices are between one percent and 1.5 percent per month. The associated risk is around seven percent for the non-U.S. countries and somewhat lower for the U.S. itself, which is due to the fact that all returns are based on indices denominated in dollars. The emerging markets are more volatile than the G5 countries, as can be seen from the standard deviations of the returns, which are always higher for the emerging markets than for the G5 countries. Due to the fact that we have emerging markets indices for regions rather than for individual countries, the standard deviations are not extremely high though, never exceeding ten percent per month. The variation in the mean returns also appears to be higher for the emerging markets, since the mean return is almost two percent for Latin America and only 0.5 percent for Asia.

Finally, Table 1 presents the average correlation of each index with the G5 countries and with the emerging markets, where the correlation of each index with itself is excluded from the average. Not surprisingly, the highest correlations are found between the G5 countries. The correlations between the emerging markets are about two-third of the correlations between the G5 countries, and the correlations between the emerging markets and the G5 countries are still lower.

4 Portfolios based on the G5 countries

In order to show the effects of estimation risk, Table 2 presents optimal portfolios for the G5 countries for three different sample periods and for different levels of the actual risk aversion γ . The first column of Panel A gives the mean-variance efficient portfolio for a risk-averse agent with $\gamma = 12$, based on the entire sample period of January 1974 until December 1998. This portfolio is located near the Global Minimum Variance (GMV) portfolio and is therefore not very susceptible to estimation risk in the mean returns. This is also evident when comparing these portfolio weights with the ones in the second column, which are the ones based on the pseudo risk aversion α^* and thus incorporate estimation risk. The differences in optimal portfolio weights appear to be relatively small in this case.

The next columns of Panel A show similar portfolio weights for two shorter sample periods. The differences in the portfolio weights are most profound in the last and shortest sample period, January 1994 until December 1998. The biggest impact of the estimation risk is on the weights for Japan and the U.S., where the adjustment for estimation risk amounts to 15 and 25 percent absolute change, respectively.

Although the biggest adjustment in terms of portfolio weights occurs for the shortest sample period, Panel B of Table 2 shows that the difference between the actual and the pseudo risk-aversion is actually the smallest for the shortest sample period. From Equation (7), this is due to the differences in the efficient set constants for the different sample periods, since the effect of the sample size T is such that the difference increases when the sample size decreases. Indeed, as can be seen from the last three lines of Panel A, the differences between the efficient set constants for the different sample periods is such that the change in the $A/(AC - B^2)$ term exceeds the change in T . This shows the relative importance of the curvature of the frontier for

the adjustments that have to be made in the optimal portfolios in order to account for estimation risk.

Panel B of Table 2 also shows the difference in expected returns for the portfolios based on the actual risk aversion γ and the pseudo risk-aversion α^* . Here we see that the differences in terms of expected return increase as the sample size decreases. Finally, the gain in utility, δ , increases as the sample period decreases and as the risk aversion decreases. For the longest sample period, which covers 25 years of monthly data, and a risk aversion $\gamma = 12$, the difference in utility translates into an equivalent risk premium of 0.046 percent per month, or about six basis points per year. For the shortest sample period and a risk aversion $\gamma = 2$, this increases to a sizable 0.559 percent per month, or 6.7 percent annually. This reflects the fact that uncertainty in the mean returns becomes more important for lower risk aversions and for shorter sample periods.

In summary, the results show that there can be sizable adjustments in portfolio weights for estimation risk. This is also reflected in the gain in utility which, for a risk aversion of 2, can be as high as 6.7 percent per year for the most recent sample period of five years. The differences between the different samples are not just due to the length of the sample period, but are also affected by the fact that the estimates of the efficient set constants are different for the different sample periods. Because mean returns are especially important for investors with low risk aversions, we find that the effects of estimation error increase when the risk aversion decreases.

5 Including emerging markets in international portfolios

The previous section shows the relative importance of the combined effects of a decrease in sample size, the actual risk aversion, uncertainty in mean returns, and the curvature of the mean-variance frontier on the adjustments that have to be made in the optimal portfolios in order to account for estimation risk. From the summary statistics in Table 1 it follows that the uncertainty in the returns on emerging markets is higher than in the returns on the G5 countries. This confirms one of the stylized facts of emerging markets returns as described in, for instance, Harvey (1995), Bekaert and Harvey (1997), and DeRoos, Nijman, and Werker (2000), who show that both the

variance of the returns, as well as the cross-sectional variability in the mean returns is much higher for emerging markets than for developed markets. In addition, the sample period for which data for these markets are available, is much shorter than for the G5 countries. Also, when looking at Equation (7), K increases from 5 to 8, which will have an added effect on the adjustment in the optimal portfolio as well. Therefore, when including emerging markets in the investment opportunity set, we may expect the effects of estimation risk to be even more pronounced than in case of the G5 countries only.

In terms of the utility gain δ , Panel B of Table 2 showed that for the G5 countries the gain is about four times higher for the period January 1989 until December 1998, than for the longer period January 1974 until December 1998. The emerging markets data are available since January 1989 only, implying that we should use this period for the G5 countries as a benchmark. From Table 3, for a risk aversion $\gamma = 12$, the expected loss increases from 0.049 percent for the G5 countries, to 0.101 percent when the emerging markets are included as well, i.e., the gain in utility is two times as high as for the G5 countries only. For the shortest sample period, the expected loss δ is about 50 percent higher when the emerging markets are included relative to the case of the G5 countries only. The resulting difference in utility translates into an equivalent premium of 0.77 percent per month, or about nine percent per year for the shortest sample period, when the risk aversion is $\gamma = 2$.

As the first panel of Table 3 shows, both the actual and the pseudo risk-aversion result in portfolios that have big short positions, especially for the short sample period. Therefore, Table 4 also shows the effects of estimation risk on the portfolios for the G5 countries and the emerging markets when there are short sales restrictions. When the ten-year period of January 1989 until December 1998 is used to calculate the optimal portfolio for a risk aversion $\gamma = 6$, use of the actual risk aversion yields a portfolio that only invests in the U.S., in Germany and in Latin America. For all other countries the short sales constraints are binding. When estimation risk is taken into account, the optimal portfolio is located on a different segment of the mean-variance frontier, and now additional positions are taken in the U.K. and the Middle East as well, mainly at the expense of the position in the U.S. market. For the shorter period January 1994 until December 1998, we even see that incorporating estimation risk shifts the portfolio from a 100% investment in the U.S. to a portfolio that also invests in Germany and the U.K. The finding that no position is taken in the emerging markets is in line with the

result in DeRoos, Nijman, and Werker (2000) that there are no significant diversification benefits from emerging markets in recent years when short sales constraints are taken into account.

Although in terms of portfolio weights the effects of estimation risk are stronger when there are short sales constraints, the second panel of Table 4 shows that the effects on expected portfolio return and on the utility gain δ are much less pronounced than in Table 3. This finding is a result of the fact that the mean-variance frontier is limited and diversification benefits are smaller when there are short sales constraints.

6 Time-varying expected returns

There is ample evidence available that stock returns can be predicted from common instruments such as the short term interest rate, the default spread, and the dividend yield on the market portfolio (see, e.g., Ferson and Harvey (1999)). When implementing these predictabilities in forming efficient portfolios, an often encountered problem is that the optimal portfolio strategy induces a lot of variability in portfolio weights. Due to transaction costs for instance, large variations in portfolio weights can be cumbersome. To the extent that the predictability in stock return is affected by estimation risk, the variability in portfolio weights may be diminished once estimation risk is explicitly accounted for in the optimal portfolio. The purpose of this section is to use our adjustment for estimation risk when implementing conditional portfolio strategies.

Section 2.3 shows how the pseudo risk-aversion α_{t-1}^* should be optimally chosen in case returns are predictable from a set of instruments z_{t-1} . We use as instruments a constant; the short term U.S. risk free interest rate at the beginning of the month measured by the one-month TBill-rate; the term spread, which is the spread between the yield on the ten-year U.S. treasury note and the short term U.S. interest rate; the default spread, which is the yield spread between Moody's Baa and Aaa rate U.S. bonds; and the spread between the lagged dividend yield on the world portfolio and the short term U.S. interest rate. These instruments are the same as in DeSantis and Gerard (1997) for instance², and are often used in empirical studies to predict stock returns and are known to have some predictive power. Here, these instruments are used to predict returns on the G5 countries. Following

²Except for the dividend yield, which in our case is the yield on the MSCI World index.

the setup in Section 2.3, we assume that expected returns are a linear function of the instruments, whereas variances are constant over time.

Table 5 summarizes the results of the mean-variance efficient portfolio weights for the G5 countries when returns are predicted from the five instruments (including a constant) described above. The results in this table are based on the entire sample period, which contains 300 observations. The last column of Table 5 presents the R^2 's of the predictive regressions of each of the five country returns on the instruments. The R^2 is always lower than five percent, and typically lower than the R^2 's reported by for instance Ferson and Harvey (1999). However, they use U.S. instruments to explain domestic stock portfolios, whereas we use both U.S. and global (the dividend yield) instruments to explain country returns.

The first two columns show the mean and standard deviation of the unadjusted conditional mean-variance portfolio weights that are based on a risk aversion parameter $\gamma = 12$.³ The standard deviations reflect the common finding that implementing conditioning information leads to large variations in the optimal portfolio weights. Even though the risk aversion is relatively high, implying that the portfolio should not be too sensitive to variation in expected returns, the standard deviation of the portfolio weights for the non-U.S. countries is about 35%. Since the pseudo risk aversion α^* takes into account the estimation risk in the predictive regressions, accounting for estimation risk may result in different and less variable portfolio weights. Indeed, the third and fourth column of Table 5 show that the means and standard deviations of the adjusted conditional mean-variance portfolio weights are different from the unadjusted ones in the first two columns.

The adjusted mean portfolio weights for Japan and the U.S. are less extreme than the unadjusted ones, and, more importantly, the standard deviations are about half the ones of the unadjusted weights. This is also shown by the fifth column, which shows the percentage reduction in the variance of the portfolio weights that results from taking estimation risk into account. Here we see that on average there is about 70 percent variance reduction in the weights. This suggests that the estimation risk in the predictive regressions is substantial and that accounting for this risk leads to conditional mean-variance portfolio weights that are much less variable than a straightforward implementation of the predictive regressions would suggest.

³Results for other risk aversions are very similar and can be obtained from the authors upon request.

7 Uncertainty in covariances

As a final part of the analysis we wish to address the effect of estimation risk in the (co)variances, to see whether this is indeed small relative to the effect of estimation error in the expected returns. To this end, we simulate a set of returns and analyze the loss in utility that occurs when calculating optimal portfolios based on either the true or the estimated means and (co)variances. Specifically, we use the sample means and (co)variances of the G5 countries as the actual expected returns and co(variances) of our assets. From this we simulate a sample of T returns, assuming that the asset returns are normally distributed. For each simulation we then calculate the loss in expected utility

$$\begin{aligned}
L(\gamma; \hat{\mu}_T, \Sigma) &= \left\{ w^{*'} \mu - \hat{w}(\gamma; \hat{\mu}_T, \Sigma)' \mu \right\} \\
&\quad - \frac{1}{2} \gamma \left\{ w^{*'} \Sigma w^* - \hat{w}(\gamma; \hat{\mu}_T, \Sigma)' \Sigma \hat{w}(\gamma; \hat{\mu}_T, \Sigma) \right\}, \\
L(\gamma; \mu, \hat{\Sigma}_T) &= \left\{ w^{*'} \mu - \hat{w}(\gamma; \mu, \hat{\Sigma}_T)' \mu \right\} \\
&\quad - \frac{1}{2} \gamma \left\{ w^{*'} \Sigma w^* - \hat{w}(\gamma; \mu, \hat{\Sigma}_T)' \Sigma \hat{w}(\gamma; \mu, \hat{\Sigma}_T) \right\}, \\
&\text{and} \\
L(\gamma; \hat{\mu}_T, \hat{\Sigma}_T) &= \left\{ w^{*'} \mu - \hat{w}(\gamma; \hat{\mu}_T, \hat{\Sigma}_T)' \mu \right\} \\
&\quad - \frac{1}{2} \gamma \left\{ w^{*'} \Sigma w^* - \hat{w}(\gamma; \hat{\mu}_T, \hat{\Sigma}_T)' \Sigma \hat{w}(\gamma; \hat{\mu}_T, \hat{\Sigma}_T) \right\},
\end{aligned}$$

where w is calculated according to (2), based on either the actual expected returns and (co)variances μ and Σ , resulting in the optimal portfolio w^* or based on the estimates $\hat{\mu}_T$ and $\hat{\Sigma}_T$, that are obtained from the T simulated returns.

Table 6 shows the averages of the losses in the expected utility over 10,000 simulations, which can be interpreted as the measure δ :

$$\begin{aligned}
\delta(\gamma; \hat{\mu}_T, \Sigma) &= E[L(\gamma; \hat{\mu}_T, \Sigma)], \\
\delta(\gamma; \mu, \hat{\Sigma}_T) &= E[L(\gamma; \mu, \hat{\Sigma}_T)], \\
\delta(\gamma; \hat{\mu}_T, \hat{\Sigma}_T) &= E[L(\gamma; \hat{\mu}_T, \hat{\Sigma}_T)].
\end{aligned}$$

These measures show the relative importance of estimation error in the expected returns and in the (co)variances of the returns. It is obvious from

this Table 6 that the effects of estimation risk is much more relevant for the expected returns than for the covariances. For every risk aversion and sample size in Table 6, the expected loss that is due to uncertainty in the expected returns, is at least six times as high as the loss that is due to uncertainty in the covariances. The third line of each panel in Table 6 shows the combined effect of estimation error in the means and the covariances. From these lines we see that there is also an interaction effect of the estimation errors which results in a total expected loss that in most cases exceeds the sum of the individual effects of the estimation errors in the means and the covariances.

In terms of loss in expected utility, the uncertainty in expected returns becomes less important as the risk aversion increases, whereas the uncertainty in the covariances becomes more important. Naturally, this reflects the fact that as the risk aversion increases, the interest is more in the variance of the portfolio return than in the expected portfolio return. As the risk aversion is 1, the effect of uncertainty in expected returns is about 150 times larger than the effect of uncertainty in the covariances. As the risk aversion increases to 10, this ratio decreases to 6. This ratio appears to be independent of the sample size, although the magnitude of δ clearly does depend on the sample size.

Clearly, the simulations show that the loss in expected utility from uncertainty in covariances is small. This is common knowledge in the literature and justifies our approach which focuses on the uncertainty in expected returns only. Although the relative importance of uncertainty in the expected returns compared with the uncertainty in the covariances decreases as the risk aversion increases, the loss in expected utility caused by uncertainty in the covariances appears to be small in all cases.

8 Summary and conclusions

This paper proposes an adjustment in mean-variance portfolio weights to incorporate estimation risk caused by uncertainty in expected security returns. Assuming that asset returns are homoskedastic and normally distributed, the adjustment amounts to using a pseudo risk-aversion rather than the agent's actual risk aversion. This pseudo risk-aversion is always higher than the actual risk aversion and the difference between the two depends on the number of assets under consideration, the sample size, and the efficient set constants. As is to be expected, the difference between the pseudo risk-aversion and the

actual risk aversion is increasing in the number of assets included in the portfolio and decreasing in the sample size. When returns are predictable from a set of observed instruments, the adjustment also depends on the values taken by the instruments.

Applying the adjustment to international portfolios, we show that the adjustments are nontrivial for the G5 country portfolios and that they are even more important when emerging markets are included. We also show that, in case of time-varying expected country returns, our adjustment implies a significantly smaller variability in portfolio weights.

Simulations suggest that uncertainty in the expected returns is much more important than uncertainty in the covariances, for all sample sizes and for all risk aversions considered. As can be expected, the uncertainty in covariances becomes more important as the risk aversion increases, although the magnitude of the loss in expected utility that results from this uncertainty remains small. Future research plans to take this uncertainty into account as well.

A Derivation of the adjustment factor

From Equation (5) and (6), the expected loss in expected utility from using \hat{w} instead of w is

$$\delta = E \left[(w^*{}'\mu - \hat{w}(\alpha)'\mu)' - \frac{1}{2}\gamma (w^*{}'\Sigma w^* - \hat{w}(\alpha)'\Sigma \hat{w}(\alpha)) \right].$$

The problem is to choose α in order to minimize this loss. This comes down to maximizing

$$\max_{\alpha} E [\hat{w}(\alpha)'\mu] - \frac{1}{2}\gamma E [\hat{w}(\alpha)'\Sigma \hat{w}(\alpha)].$$

Since the returns R_t are normally distributed with mean vector μ and covariance matrix Σ , it follows that $\hat{\mu} \sim N(\mu, \Sigma/T)$. We need to consider two quantities: $E[\hat{w}(\alpha)]$ and $E[\hat{w}(\alpha)'\Sigma \hat{w}(\alpha)]$. First, observe that

$$\begin{aligned} E [\hat{w}(\alpha)] &= E \left[\alpha^{-1} \Sigma^{-1} \left(\hat{\mu} - \frac{\hat{B} - \alpha}{A} \iota \right) \right] \\ &= \alpha^{-1} \Sigma^{-1} \left(\mu - \frac{B - \alpha}{A} \iota \right). \end{aligned}$$

For the variance term, we find

$$\begin{aligned}
& E[\hat{w}(\alpha)' \Sigma \hat{w}(\alpha)] \\
&= E \left[\alpha^{-2} \left(\hat{\mu} - \frac{\hat{B} - \alpha}{A} \right)' \Sigma^{-1} \left(\hat{\mu} - \frac{\hat{B} - \alpha}{A} \right) \right] \\
&= \alpha^{-2} \left(\frac{K}{T} + \mu' \Sigma^{-1} \mu \right) - \alpha^{-2} \left(\frac{1}{T} + \frac{B^2}{A} \right) + \frac{1}{A}.
\end{aligned}$$

Together, these imply that we have to maximize

$$\begin{aligned}
& \alpha^{-1} \left(C - B \frac{B - \alpha}{A} \right) - \frac{1}{2} \gamma \left(\alpha^{-2} \left(\frac{K - 1}{T} + C \right) - \alpha^{-2} \left(\frac{B^2}{A} \right) + \frac{1}{A} \right) \\
&= \alpha^{-1} \left(C - \frac{B^2}{A} \right) - \alpha^{-2} \frac{1}{2} \gamma \left(\frac{K - 1}{T} + C - \frac{B^2}{A} \right) + \frac{1}{A} \left(B - \frac{1}{2} \gamma \right).
\end{aligned}$$

Maximizing and solving for α gives

$$\alpha^* = \gamma \left(1 + \frac{A}{AC - B^2} \frac{K - 1}{T} \right).$$

The associated gain in utility is equal to

$$\delta(\alpha^*) = \frac{1}{2\alpha^*} \left(\frac{A}{AC - B^2} \right) \left(\frac{K - 1}{T} \right)^2.$$

B The adjustment factors in case returns are predictable

Next consider the case where the returns R_t can be predicted from a set of L instruments (which may include a constant) z_t :

$$R_t = \beta z_{t-1} + \varepsilon_t,$$

with $\Omega_{\varepsilon\varepsilon} = \text{Var}[\varepsilon_t]$. This can be rewritten as

$$R_t = (z'_{t-1} \otimes I) b + \varepsilon_t,$$

with $b = \text{vec}(\beta)$. Notice that for $z_{t-1} = z_0$

$$\hat{\mu}_0 = (z'_0 \otimes I) \hat{b},$$

implying that, conditionally on the z_{t-1} ,

$$\begin{aligned} Var[\hat{\mu}_0] &= Var \left[(z'_0 \otimes I) \hat{b} \right] \\ &= (z'_0 \otimes I) \left\{ \left(\sum_{t=1}^T z_{t-1} z'_{t-1} \right)^{-1} \otimes \Omega_{\varepsilon\varepsilon} \right\} (z_0 \otimes I) \\ &= T^{-1} \left(z'_0 \left(T^{-1} \sum_{t=1}^T z_{t-1} z'_{t-1} \right)^{-1} z_0 \right) \Omega_{\varepsilon\varepsilon}. \end{aligned}$$

Similar to the unconditional case, the problem to solve comes down to

$$\max_{\alpha} E_{t-1} [\mu'_0 \hat{w}(\alpha)_0] - \frac{1}{2} \gamma E [\hat{w}(\alpha)'_0 \Omega_{\varepsilon\varepsilon} \hat{w}(\alpha)_0].$$

For this, we need to consider two quantities: $E[\hat{w}(\alpha)_0]$ and $E[\hat{w}(\alpha)'_0 \Omega_{\varepsilon\varepsilon} \hat{w}(\alpha)_0]$.

First, observe that

$$\begin{aligned} E[\hat{w}(\alpha)_0] &= E \left[\alpha^{-1} \Omega_{\varepsilon\varepsilon}^{-1} \left(\hat{\mu}_0 - \frac{\hat{B}_0 - \alpha}{A} \iota \right) \right] \\ &= \alpha^{-1} \Omega_{\varepsilon\varepsilon}^{-1} \left(\mu_0 - \frac{B_0 - \alpha}{A} \iota \right). \end{aligned}$$

For the variance term, we find

$$\begin{aligned} E[\hat{w}(\alpha)'_0 \Omega_{\varepsilon\varepsilon} \hat{w}(\alpha)_0] &= \alpha^{-2} E[\hat{\mu}'_0 \Omega_{\varepsilon\varepsilon}^{-1} \hat{\mu}_0] - 2\alpha^{-2} E \left[\hat{B}_0^2 \frac{1}{A} \right] \\ &\quad + 2\alpha^{-2} E \left[\frac{\alpha}{A} \hat{\mu}'_0 \Omega_{\varepsilon\varepsilon}^{-1} \iota \right] + \alpha^{-2} E \left[\left(\frac{\hat{B}_0 - \alpha}{A} \right)^2 \iota' \Omega_{\varepsilon\varepsilon}^{-1} \iota \right]. \end{aligned}$$

Using the trace-operator we get from

$$\hat{\mu}_0 \sim N \left(\mu_0, \left\{ z'_0 \left(T^{-1} \sum_{t=1}^T z_{t-1} z'_{t-1} \right)^{-1} z_0 \right\} \Omega_{\varepsilon\varepsilon} \right),$$

that

$$E[\hat{\mu}'_0 \Omega_{\varepsilon\varepsilon}^{-1} \hat{\mu}_0] = E[(\hat{\mu}_0 - \mu_0)' \Omega_{\varepsilon\varepsilon}^{-1} (\hat{\mu}_0 - \mu_0)] + C_0$$

$$\begin{aligned}
&= E \left[\left(\widehat{b} - b \right)' \left(z_0 z_0' \otimes \Omega_{\varepsilon\varepsilon}^{-1} \right) Var[\widehat{b}] Var[\widehat{b}]^{-1} \left(\widehat{b} - b \right) \right] + C_0 \\
&= \frac{K}{T} \left(z_0' \left(T^{-1} \sum_{t=1}^T z_{t-1} z_{t-1}' \right)^{-1} z_0 \right) + C_0.
\end{aligned}$$

Similarly,

$$\begin{aligned}
E \left[\widehat{B}_0^2 \frac{1}{A} \right] &= E \left[\frac{\iota' \Omega_{\varepsilon\varepsilon}^{-1} \widehat{\mu}_0 \widehat{\mu}_0' \Omega_{\varepsilon\varepsilon}^{-1} \iota}{A} \right] \\
&= E \left[\frac{B_0^2}{A} + T^{-1} \left(z_0' \left(T^{-1} \sum_{t=1}^T z_{t-1} z_{t-1}' \right)^{-1} z_0 \right) \right],
\end{aligned}$$

implying that the variance term reduces to

$$\begin{aligned}
&E [\widehat{w}(\alpha)'_0 \Omega_{\varepsilon\varepsilon} \widehat{w}(\alpha)_0] \\
&= \alpha^{-2} \left(\frac{K}{T} D_0 + C_0 \right) - \alpha^{-2} \left(\frac{B_0^2}{A} + \frac{D_0}{T} \right) + \frac{1}{A},
\end{aligned}$$

with

$$D_0 = z_0' \left(T^{-1} \sum_{t=1}^T z_{t-1} z_{t-1}' \right)^{-1} z_0.$$

Thus, we have to maximize

$$\alpha^{-1} \left(C_0 - \frac{B_0^2}{A} \right) + \frac{B_0}{A} - \frac{1}{2} \gamma \left(\alpha^{-2} (K D_0 + C_0) - \alpha^{-2} \left(\frac{B_0^2}{A} + D_0 \right) + \frac{1}{A} \right).$$

Maximizing with respect to α and solving for α gives

$$\alpha_0^* = \gamma \left(1 + \frac{(K-1)}{T} D_0 \frac{A}{A C_0 - B_0^2} \right).$$

References

- Alexander, G.J., and Resnick, B.G., 1985, "More on Estimation Risk and Simple Rules for Optimal Portfolio Selection", *Journal of Finance*, 40, p.125-133.
- Bekaert, G., and Harvey, C.R., 1997, "Emerging Equity Market Volatility", *Journal of Financial Economics*, 43, p.29-77.
- Barry, Ch.B., 1974, "Portfolio Analysis under Uncertain Means, Variances, and Covariances", *Journal of Finance*, 29, p.515-522.
- Chen, S. and Brown, S.J., 1983, "Estimation Risk and Simple Rules for Optimal Portfolio Selection", *Journal of Finance*, 38, p.1087-1093.
- DeRoos, F.A., Nijman, Th.E., and Werker, B.J.M., 2000, "Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets", *Journal of Finance*, forthcoming.
- DeSantis, G., and Gerard, B., 1997, "International Asset Pricing and Portfolio Diversification with Time-Varying Risk", *Journal of Finance*, 52, p.1881-1912.
- Dybvig, Ph.H., 1984, "Short Sales Restrictions and Kinks on the Mean Variance Frontier", *Journal of Finance*, 39, p.239-244.
- Ferson, W.E., and Harvey, C.R., 1999, "Conditioning Variables and the Cross Section of Stock Returns", *Journal of Finance*, 54, p.1325-1360.
- Harvey, C.R., 1995, "Predictable Risk and Returns in Emerging Markets", *Review of Financial Studies*, 8, p.773-816.
- Jobson, J.D., Korkie, B., and Ratti, V., 1979, "Improved Estimation for Markowitz Portfolios using James-Stein Type Estimators", *Proceedings of the American Statistical Association, Business and Economics Statistics Section*, p.279-284.
- Jorion, Ph., 1985, "International Portfolio Diversification with Estimation Risk", *Journal of Business*, 58, p.259-278.

- Jorion, Ph., 1986, "Bayes-Stein Estimation for Portfolio Analysis", *Journal of Financial and Quantitative Analysis*, 21, p.279-292.
- Jorion, Ph., 1991, "Bayesian and CAPM Estimators of the Means: Implications for Portfolio Selection", *Journal of Financial Economics*, p.717-727.
- Klein, R.W., and Bawa, V.S., 1976, "The Effect of Estimation Risk on Optimal Portfolio Choice", *Journal of Financial Economics*, 3, p.215-231.
- Markowitz, H.M., 1959, "Portfolio Selection: Efficient Diversification of Investments", Wiley, New York.
- Markowitz, H.M., 1987, "Mean-Variance Analysis in Portfolio Choice and Capital Markets", Basil Blackwell, Cambridge, Massachusetts.

Table 1: Summary statistics

The table contains summary statistics of monthly dollar denominated returns for the G5 countries and three emerging markets indices. Means and standard deviations are in percentages. The correlations are the average correlation of each country or region with the G5 countries and the average correlation with the emerging markets, excluding the correlation of each country or region with itself. The G5 indices are the MSCI indices, the emerging market indices are the IFC Investable indices.

<i>G5 Countries, 01/74-12/98</i>				
	<i>mean</i>	<i>stdev</i>	<i>corr(G5)</i>	<i>corr(Em)</i>
France	1.37	6.85	0.592	0.328
Germany	1.34	5.89	0.522	0.288
Japan	1.11	6.70	0.384	0.229
UK	1.49	7.22	0.584	0.253
USA	1.25	4.49	0.466	0.328
<i>Emerging Markets, 01/89-12/98</i>				
	<i>mean</i>	<i>stdev</i>	<i>corr(G5)</i>	<i>corr(Em)</i>
Latin America	1.94	9.87	0.228	0.323
Asia	0.54	7.90	0.385	0.370
Middle East+Europe	0.85	9.38	0.243	0.313

Table 2: Efficient portfolios of the G5 countries, incorporating estimation risk

The table presents the effects of estimation risk on optimal portfolios for different sample periods and different levels of risk aversion. Panel A shows mean-variance efficient portfolio weights for an agent with actual risk aversion $\gamma = 12$ for the three sample periods, with and without the correction for estimation risk. $w(\gamma)$ is the efficient portfolio based on the actual risk aversion and $w(\alpha)$ is the efficient portfolio based on the pseudo risk aversion. Panel B shows the differences between optimal portfolios for the three sample periods and for three different levels of the actual risk aversion γ . α is the pseudo risk-aversion. $E[r_t^p]$ gives the estimated mean portfolio return on the portfolios with and without a correction for estimation risk. δ gives the expected loss in utility due to estimation risk. All results are based on monthly dollar denominated returns on the MSCI indices for the G5 countries.

Panel A: Optimal portfolio weights, $\gamma = 12$						
	01/74-12/98		01/89-12/98		01/94-12/98	
	$w(\gamma)$	$w(\alpha)$	$w(\gamma)$	$w(\alpha)$	$w(\gamma)$	$w(\alpha)$
France	-0.036	-0.048	0.035	0.037	-0.056	-0.077
Germany	0.254	0.241	0.114	0.112	0.000	0.057
Japan	0.149	0.189	-0.223	-0.126	-0.399	-0.235
U.K.	0.032	-0.014	0.155	0.128	0.481	0.504
U.S.	0.600	0.632	0.927	0.848	0.974	0.751
$A/(AC - B^2)$	361.4		16.41		7.56	
Panel B: Comparing different risk aversions						
	01/74-12/98		01/89-12/98		01/94-12/98	
	γ	α	γ	α	γ	α
$risk.av.$	12.0	69.9	12.0	18.6	12.0	18.05
$E[r_t^p](\%)$	1.25	1.23	1.91	1.74	2.55	2.18
$\delta(\%)$		0.046		0.049		0.093
$risk.av.$	6.0	34.5	6.0	9.3	6.0	9.0
$E[r_t^p](\%)$	1.28	1.24	2.42	2.07	3.65	2.91
$\delta(\%)$		0.092		0.098		0.186
$risk.av.$	2.0	11.6	2.0	3.1	2.0	3.0
$E[r_t^p](\%)$	1.37	1.25	4.46	3.34	8.06	5.84
$\delta(\%)$		0.276		0.295		0.559

Table 3: Estimation risk for the G5 countries plus emerging markets

The table presents the effects of estimation risk on optimal portfolios for different sample periods and different levels of risk aversion. Panel A shows mean-variance efficient portfolio weights for an agent with actual risk aversion $\gamma = 12$ for the three sample periods, with and without the correction for estimation risk. $w(\gamma)$ is the efficient portfolio based on the actual risk aversion and $w(\alpha)$ is the efficient portfolio based on the pseudo risk aversion. Panel B shows the differences between optimal portfolios for the three sample periods and for three different levels of the actual risk aversion γ . $E[r_t^p]$ gives the estimated mean portfolio return on the portfolios with and without a correction for estimation risk. δ gives the expected loss in utility due to estimation risk. All results are based on monthly dollar denominated returns on the MSCI indices for the G5 countries and on the IFC Investable indices for the emerging markets.

<i>Panel A: Optimal portfolio weights, $\gamma = 12$</i>				
	<i>01/89-12/98</i>		<i>01/94-12/98</i>	
	$w(\gamma)$	$w(\alpha)$	$w(\gamma)$	$w(\alpha)$
France	-0.055	-0.038	0.177	0.101
Germany	0.175	0.149	-0.006	0.042
Japan	-0.210	-0.093	-0.308	-0.194
U.K.	0.198	0.151	0.412	0.440
U.S.	0.962	0.863	1.579	1.276
Latin Am.	0.103	0.063	-0.205	-0.175
S.E. Asia	-0.221	-0.157	-0.400	-0.315
Middle East	0.050	0.062	-0.250	-0.174
<i>Panel B: Comparing different risk aversions</i>				
	<i>01/89-12/98</i>		<i>01/94-12/98</i>	
	γ	α	γ	α
<i>risk.av.</i>	12.0	20.5	12.0	16.3
$E[r_t^p](\%)$	2.12	1.83	4.45	3.73
$\delta(\%)$		0.101		0.129
<i>risk.av.</i>	6.0	10.3	6.0	8.2
$E[r_t^p](\%)$	2.80	2.22	7.14	5.71
$\delta(\%)$		0.202		0.258
<i>risk.av.</i>	2.0	3.4	2.0	2.7
$E[r_t^p](\%)$	5.55	3.83	17.92	13.63
$\delta(\%)$		0.606		0.773

Table 4: Estimation risk with short sales constraints

The table presents the effects of estimation risk on optimal portfolios for different sample periods and different levels of risk aversion, taking into account short sales constraints. Panel A shows mean-variance efficient portfolio weights for an agent with actual risk aversion $\gamma = 6$ for the three sample periods, with and without the correction for estimation risk. $w(\gamma)$ is the efficient portfolio based on the actual risk aversion and $w(\alpha)$ is the efficient portfolio based on the pseudo risk aversion. Panel B shows the differences between optimal portfolios for the three sample periods and for three different levels of the actual risk aversion γ . $E[r_t^p]$ gives the estimated mean portfolio return on the portfolios with and without a correction for estimation risk. δ gives the expected loss in utility due to estimation risk. All results are based on monthly dollar denominated returns on the MSCI indices for the G5 countries and on the IFC Investable indices for the emerging markets.

<i>Panel A: Optimal portfolio weights, $\gamma = 6$</i>				
	<i>01/89-12/98</i>		<i>01/94-12/98</i>	
	$w(\gamma)$	$w(\alpha)$	$w(\gamma)$	$w(\alpha)$
France				
Germany	0.032	0.101		0.036
Japan				
U.K.		0.101		0.180
U.S.	0.884	0.732	1.000	0.784
Latin Am.	0.084	0.013		
S.E. Asia				
Middle East		0.053		
<i>Panel B: Comparing different risk aversions</i>				
	<i>01/89-12/98</i>		<i>01/94-12/98</i>	
	γ	α	γ	α
<i>risk.av.</i>	12.0	45.2	12.0	25.0
$E[r_t^p](\%)$	1.58	1.51	1.85	1.72
$\delta(\%)$		0.102		0.072
<i>risk.av.</i>	6.0	50.2	6.0	12.5
$E[r_t^p](\%)$	1.62	1.50	1.96	1.84
$\delta(\%)$		0.225		0.144
<i>risk.av.</i>	2.0	50.2	2.0	71.9
$E[r_t^p](\%)$	1.67	1.50	1.96	1.64
$\delta(\%)$		0.641		0.146

Table 5: Estimation risk for the G5 countries using conditioning information. The table presents the effects of estimation risk on optimal portfolios for when returns are predictable. The G5 country returns are predicted from a common set of instruments. The instruments used are a constant, the short term U.S. interest rate, the U.S. term spread, the U.S. default spread, and the spread between the dividend yield on the MSCI world portfolio and the U.S. short term interest rate. The table gives the means and standard deviations of the optimal portfolios weights for the actual risk aversion $\gamma = 12$ and the pseudo risk aversion α . ΔVar gives the percentage reduction in the variance of the portfolio weights, due to using the pseudo risk aversion instead of the actual risk aversion. The last column gives the R^2 's of predictive regressions of the country returns on the instruments. The results are based on the entire sample periode, January 1974 until December 1998.

	$w(\gamma)$		$w(\alpha^*)$		ΔVar	R^2
	<i>avg</i>	<i>std</i>	<i>avg</i>	<i>std</i>		
Fra	0.05	0.40	0.04	0.20	75%	3.1%
Ger	0.11	0.35	0.14	0.18	74%	2.1%
Jap	-0.24	0.32	-0.12	0.21	57%	2.8%
U.K.	0.16	0.33	0.13	0.17	74%	3.7%
U.S.	0.93	0.13	0.80	0.06	79%	3.8%

Table 6: Simulation results for estimation errors in the expected returns and the (co)variances

The table shows the average difference in utility when the optimal portfolio is calculated from the actual expected returns and covariances or from the estimated expected returns and covariances. Returns on five assets are simulated, assuming that returns are normally distributed with means and covariances equal to those of the G5 countries. 10,000 samples with different lengths are simulated and the expected losses in utility $\delta(\gamma; \hat{\mu}, \Sigma)$, $\delta(\gamma; \mu, \hat{\Sigma})$, and $\delta(\gamma; \hat{\mu}, \hat{\Sigma})$ are calculated (in percentages).

	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$
$T = 60$				
$\delta(\gamma; \hat{\mu}, \Sigma)$ (%)	3.343	1.671	0.669	0.334
$\delta(\gamma; \mu, \hat{\Sigma})$ (%)	0.022	0.020	0.032	0.059
$\delta(\gamma; \hat{\mu}, \hat{\Sigma})$ (%)	4.365	2.191	0.900	0.493
$T = 120$				
$\delta(\gamma; \hat{\mu}, \Sigma)$ (%)	1.672	0.836	0.334	0.167
$\delta(\gamma; \mu, \hat{\Sigma})$ (%)	0.009	0.009	0.015	0.028
$\delta(\gamma; \hat{\mu}, \hat{\Sigma})$ (%)	1.913	0.961	0.396	0.219
$T = 300$				
$\delta(\gamma; \hat{\mu}, \Sigma)$ (%)	0.661	0.331	0.132	0.073
$\delta(\gamma; \mu, \hat{\Sigma})$ (%)	0.004	0.003	0.006	0.010
$\delta(\gamma; \hat{\mu}, \hat{\Sigma})$ (%)	0.702	0.352	0.145	0.080